

Optimization of MUSIC algorithm for angle of arrival estimation in wireless communications

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Abstract Smart Antennas are phased array antennas with smart signal processing algorithms used to identify the angle of arrival (AOA) of the signal, which can be used subsequently to calculate beam-forming vectors needed to track and locate the intended mobile set. This concept is called space division multiple access (SDMA) which enables a higher capacity and data rates for all modern wireless communications by focusing the antenna beam on the intended user. This enables wide coverage and very low interference and also adding new applications like location based services. Multiple Signal Classification (MUSIC) is a well-known high resolution eigen structure method, extensively used to estimate the number of signals, and their angles of arrival. In this paper we investigate the possibility of optimization of some key parameters of the MUSIC algorithm that can enhance the performance of the estimation process. This leads to an increased accuracy in determining the directions of multiple users and beam-forming (Gross, 2005).

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1. Introduction

Angle of arrival (AOA) estimation is the process of determining the direction of an incoming signal from mobile devices to the Base Transceiver Station. In this process we determine the time “phase” difference of arrival (TDOA) at individual

elements of the antenna array as shown in Fig. 1 and from these delays the angle (or direction) of the mobile devices can be calculated.

The estimation technique is done via a function which is traditionally called the pseudo spectrum PMU (θ). There are several potential approaches to define that function via: beam-forming, array correlation matrix, eigen analysis, linear prediction, minimum variance, maximum likelihood, MUSIC, root-MUSIC, and many other approaches (Schmidt, 1986).

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2. Antenna receiver model

As shown below in Fig. 2 we have uniform linear array antenna with

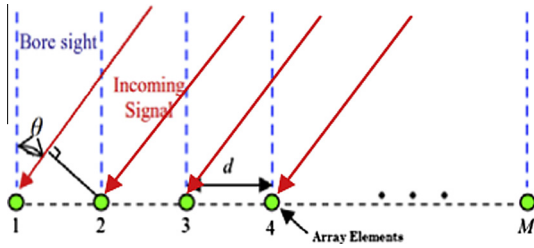


Fig. 1 Uniform linear array antenna.

- number of elements = M ,
- inter-element spacing = d ,
- number of incident signals = D ,
- number of data samples = k .

The incident signals from “ D ” users are represented in amplitude and phase at some arbitrary reference point (origin of the coordinate system) by the complex quantities S_1, S_2, \dots, S_D also white Gaussian noise added to the signals as vector (n). Directions of the incident signals represented by the steering vector $a(\theta_i)$ for i th user so we have matrix “ A ” its size $M \times D$ the first column $a(\theta_1)$ is the steering vector for the 1st user and so on, where $a(\theta_1)$ can be given as

$$a(\theta_1) = \begin{bmatrix} 1 \\ e^{j\beta d \cdot \sin(\theta_1)} \\ e^{j2\beta d \cdot \sin(\theta_1)} \\ \dots \\ e^{j(M-1)\beta d \cdot \sin(\theta_1)} \end{bmatrix}$$

where $\beta =$ incident wave number = $2\pi/\lambda$ and $d =$ inter-element spacing.

$$\begin{bmatrix} \vec{x}_1(k) \\ \vec{x}_2(k) \\ \vdots \\ \vec{x}_M(k) \end{bmatrix} = [\vec{a}(\theta_1) \quad \vec{a}(\theta_2) \quad \dots \quad \vec{a}(\theta_D)] \begin{bmatrix} S_1(k) \\ S_2(k) \\ \vdots \\ S_D(k) \end{bmatrix} + \vec{n}(k) \quad (1)$$

$$\vec{x}(k) = \vec{A} \cdot \vec{S}(k) + \vec{n}(k) \quad (2)$$

- $x(k) =$ amplitude of signal + noise in i th element \rightarrow matrix size $[M \times K]$.

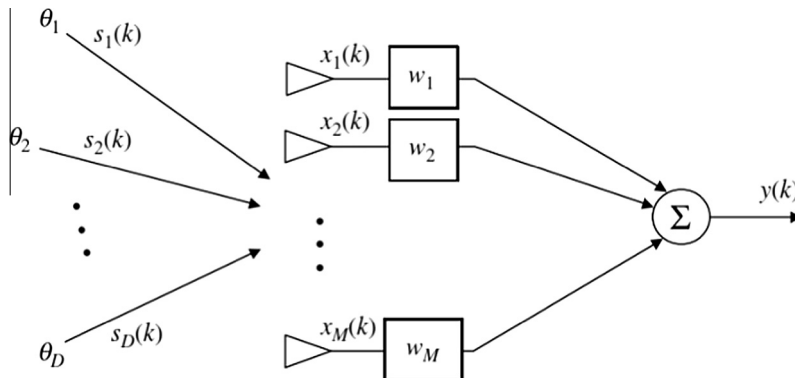


Fig. 2 Uniform linear array antenna RX model.

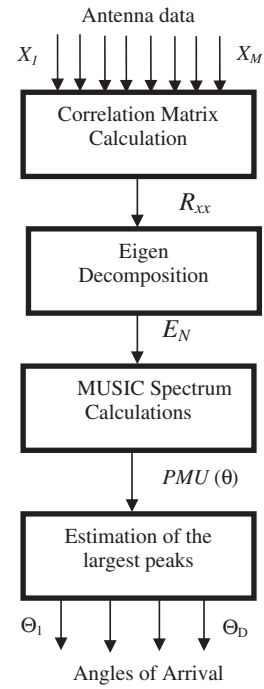


Fig. 3 MUSIC implementation flow chart.

- $S(k) =$ vector of incident signals at sample time $k \rightarrow$ matrix size $[D \times K]$.
- $n(k) =$ noise vector at each element $m \rightarrow [M \times K]$.
- $a(\theta_i) = M$ -element array steering vector $\rightarrow [M \times 1]$.
- $A = [M \times D]$ matrix of steering vectors $a(\theta_i)$.

It is initially assumed that the arriving signals are monochromatic and the number of arriving signals $D < M$. It is understood that the arriving signals are time varying and thus our calculations are based upon time snapshots of the incoming signal.

3. MUSIC algorithm

MUSIC deals with the decomposition of correlation matrix into two orthogonal matrices, signal-subspace and noise-subspace. Estimation of direction is performed from one of these subspaces, assuming that noise in each channel is highly uncorrelated. This makes the correlation matrix diagonal.

The correlation matrix is given by Gross (2005)

$$\begin{aligned}
 R_{xx} &= E[x \cdot x^H] = E[(As + n)(s^H A^H + n^H)] \\
 &= AE[s \cdot s^H]A^H + E[n \cdot n^H] \\
 &= AR_{ss}A^H + R_{nn}
 \end{aligned}
 \tag{3}$$

where H = ‘‘Hermitian’’ means conjugate transpose, E = ‘‘Expected value’’ is the statistical average, $R_{ss} = D \times D$ source correlation matrix, and $R_{nn} = M \times M$ noise correlation matrix.

The array correlation matrix has M eigen values $(\lambda_1, \lambda_2, \dots, \lambda_M)$ along with M associated eigenvectors $E = [e_1 e_2 \dots e_M]$.

If the eigen values are sorted from largest to smallest, we can divide the matrix E into two subspaces $[E_N E_S]$.

The first subspace E_N is called the noise subspace and is composed of $M - D$ eigenvectors associated with the noise.

The second subspace E_S is called the signal subspace and is composed of D eigenvectors associated with the arriving sig-

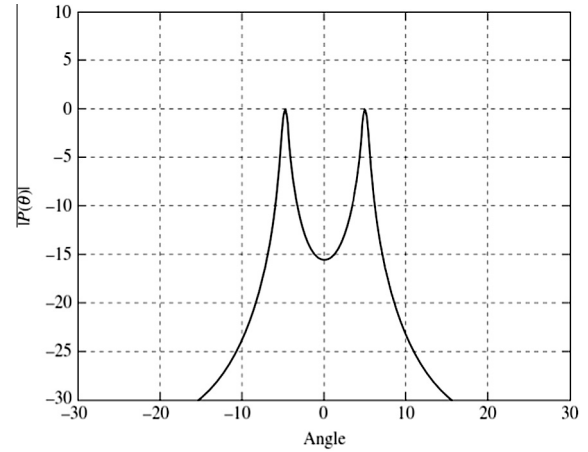


Fig. 6 Results of Ref. (Gross, 2005).

Table 1 Input data of incident signals.

Actual	Signal type	Angle (°)	SNR (dB)
Source #1	Sampled sine (1 MHz)	20	10
Source #2	Sampled sine (2 MHz)	40	10
Source #3	Sampled sine (3 MHz)	60	10

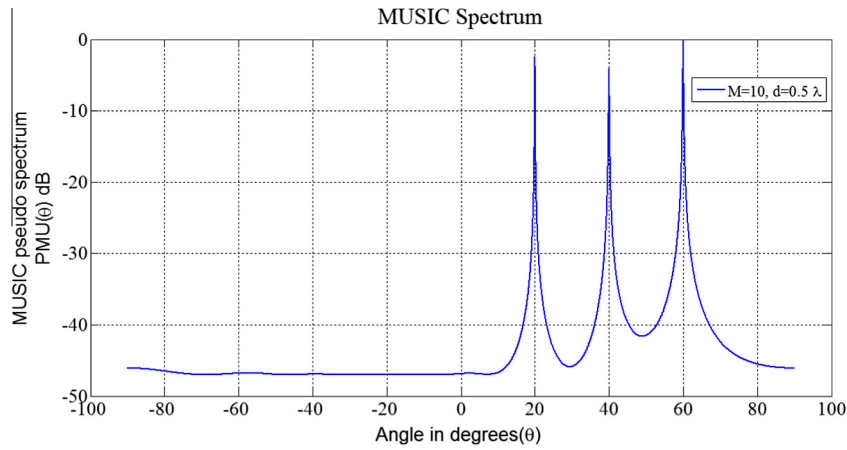


Fig. 4 MUSIC implementation.

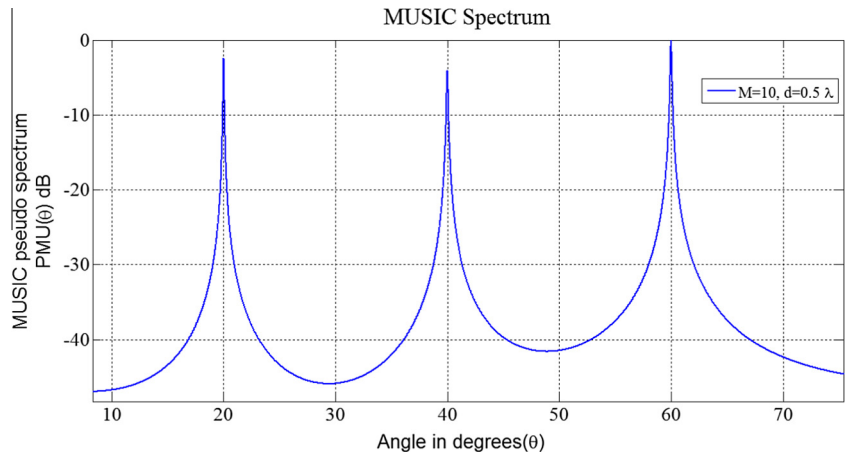


Fig. 5 Detailed view of Fig. 4 at estimated angles.

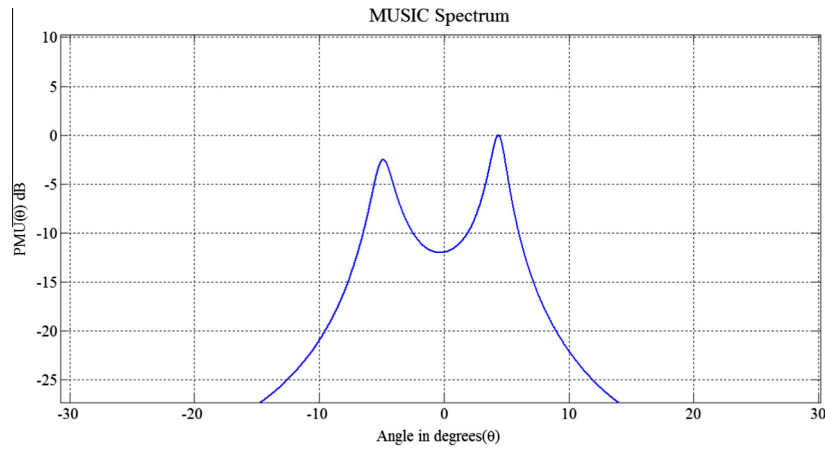


Fig. 7 MUSIC spectrum comparison with other work.

nals. The noise subspace is an $M \times (M-D)$ matrix. The signal subspace is an $M \times D$ matrix.

The noise subspace eigenvectors are orthogonal to the array steering vectors at the angles of arrival $\theta_1, \theta_2, \dots, \theta_D$. Because of this orthogonality condition, one can show that the Euclidean distance $d^2 = a(\theta)^H E_N E_N^H a(\theta) = 0$ for each and every arrival angle $\theta_1, \theta_2, \dots, \theta_D$ (Gross, 2005).

Placing this distance expression in the denominator creates sharp peaks at the angles of arrival. The MUSIC pseudo spectrum is now given as

$$PMU(\theta) = \frac{\vec{a}(\theta)^H \cdot \vec{a}(\theta)}{\vec{a}(\theta)^H \vec{E}_N \vec{E}_N^H \vec{a}(\theta)} \quad (4)$$

So, we can summarize the previous steps to estimate AOA using MUSIC as shown below in the flow chart Fig. 3.

Table 2 Input data of incident signals.

Actual	Signal type	Angle (°)	SNR (dB)
Source #1	Sampled sine (1 MHz)	10	10
Source #2	Sampled sine (2 MHz)	15	10
Source #3	Sampled sine (3 MHz)	30	10

4. Simulation

4.1. Implementation

We used MatLab software to implement the MUSIC algorithm, assuming we have the following data as shown in Table 1.

F_s = sampling frequency = 10 MHz.

Table 3 Estimated angles with different “M”.

Estimated	$M = 6$	$M = 10$	$M = 15$	$M = 20$
Source #1	Not detected	9.89°	9.98°	10.0°
Source #2	13.49°	14.94°	15.01°	14.99°
Source #3	29.94°	30.0°	30.01°	29.99°

Table 4 Input data of incident signals.

Actual	Signal type	Angle (°)	SNR (dB)
Source #1	Sampled sine (1 MHz)	10	10
Source #2	Sampled sine (2 MHz)	15	10
Source #3	Sampled sine (3 MHz)	30	10

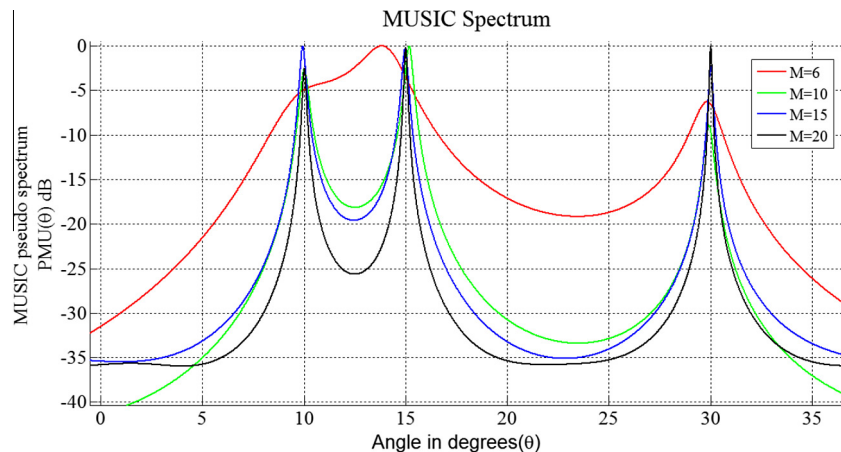


Fig. 8 MUSIC spectrum with changing “M”.

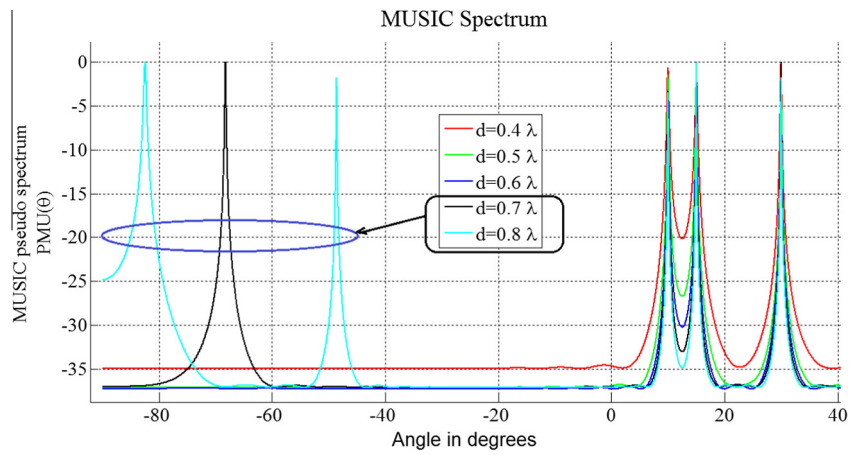


Fig. 9 MUSIC spectrum with changing “ d ”.

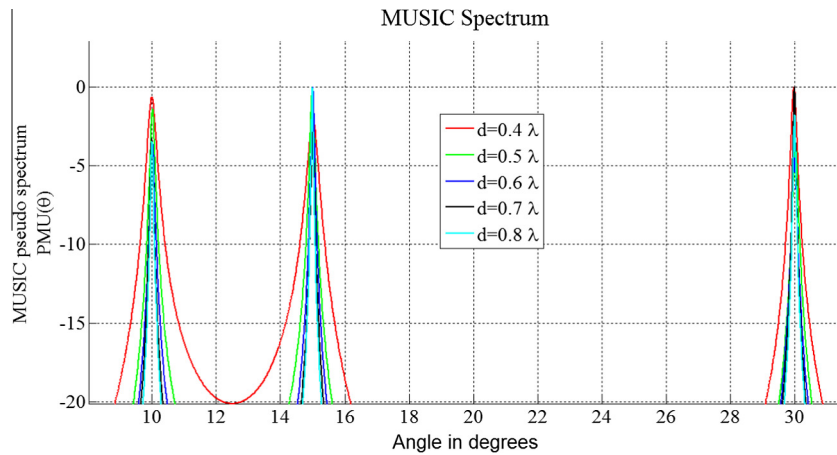


Fig. 10 Detailed view of Fig. 9 at estimated angles.

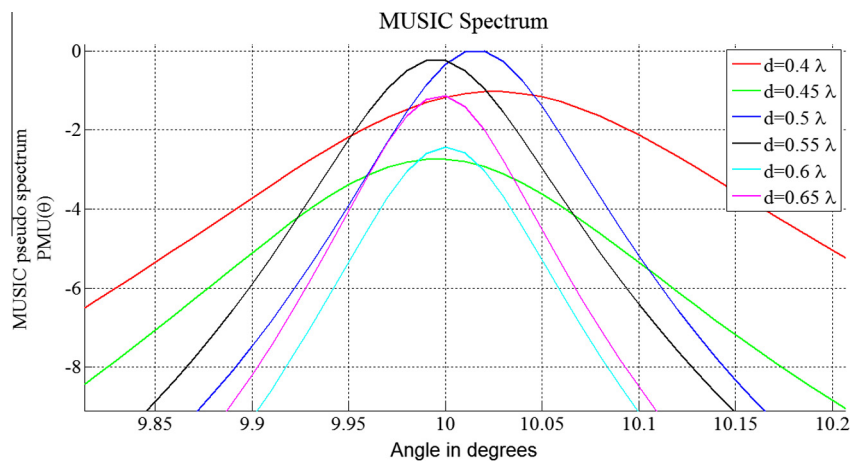


Fig. 11 Detailed view of Fig. 10 at $\theta = 10^\circ$.

k = number of data samples = 500.
 M = number of array elements = 10.
 d = inter-element spacing = 0.5λ .

Figs. 4 and 5 show estimated angles for the sources after implementation of MUSIC.

Table 5 Input data of incident signals.

Actual	Signal type	Angle (°)	SNR (dB)
Source #1	Sampled sine (1 MHz)	20	10
Source #2	Sampled sine (2 MHz)	25	10
Source #3	Sampled sine (3 MHz)	50	10

4.2. Comparison with other work

- To check the MUSIC code, which we used through our study, we calculated in Fig. 7 the PMU for some previously published case (Ref. Gross, 2005, Fig. 6). A good agreement is obviously noted.
- For two users at “+ 5°, -5°”, $k = 100$, $M = 6$, $d = 0.5\lambda$, we get the results shown in Fig. 7.
- As shown in Fig. 7 the estimated angles are + 5°, -5° as was estimated before in Gross (2005) Fig. 6.

5. Performance study

After investigation more than 100 trials of changing all parameters that can be optimized to achieve high accuracy of MU-

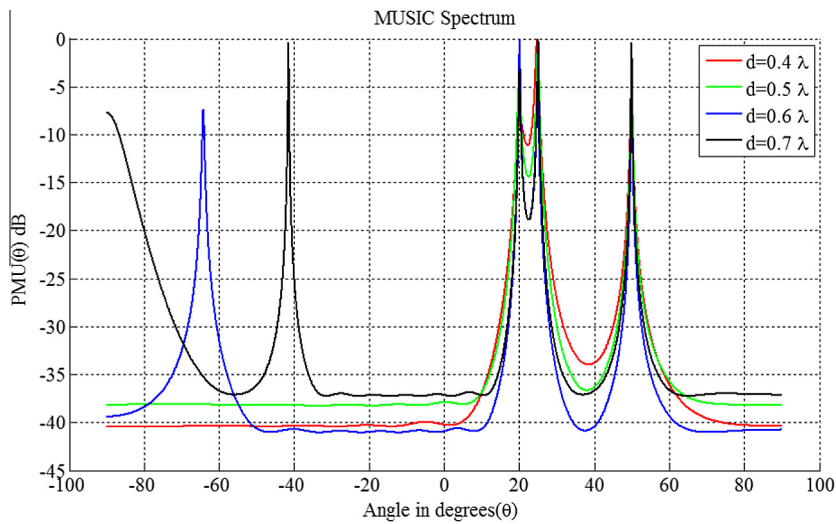


Fig. 12 MUSIC spectrum with changing “ d ”.

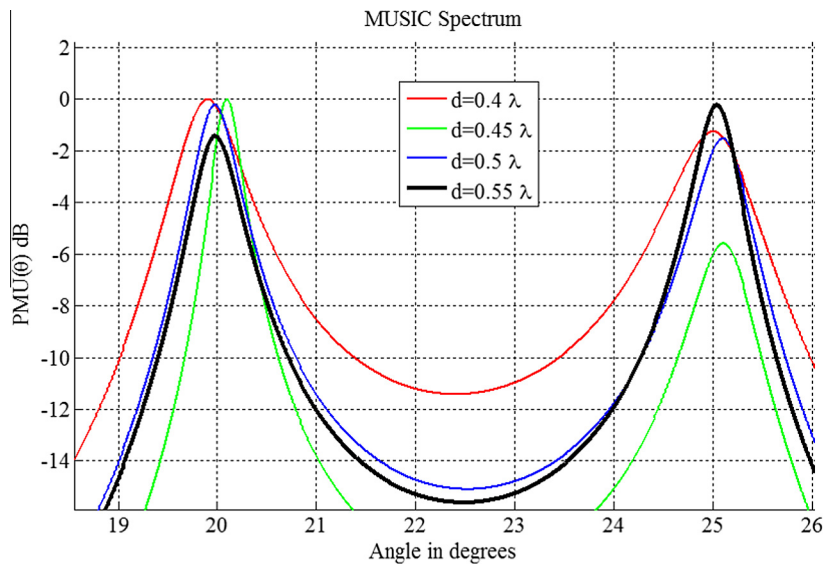


Fig. 13 Detailed view of Fig. 12.

SIC estimation are discussed in this part. You will notice from the input data to simulator as shown in Table 2 that the incident signals consider adjacent users “10°, 15°” and also non adjacent users “30°”.

5.1. MUSIC spectrum with changing number of array elements “M”

F_s = sampling frequency = 10 MHz.
 k = number of data samples = 500.
 d = inter-element spacing = 0.5λ .

■ As shown in Fig. 8 we can extract the following results about estimated angles as shown in Table 3.

1. As the number of array elements “M” increases, MUSIC spectrum peaks become sharper (high accuracy and resolution).
 2. Bad estimation is marked with $M < 10$ especially with sources #1, #2 (adjacent users).
 3. The optimum value for number of array elements is “ $M \geq 20$ ”.
- 5.2. MUSIC spectrum with changing inter-element spacing “d”

Inter-element spacing is an important factor in the design of an antenna array.

■ As d increases, the grating lobes appear which degrades the array performances.

Table 6 Input data of incident signals.

Actual	Signal type	Angle (°)
Source #1	Sampled sine (1 MHz)	10
Source #2	Sampled sine (2 MHz)	15
Source #3	Sampled sine (3 MHz)	30

Table 7 Input data of incident signals.

Actual	Signal type	Angle (°)	SNR (dB)
Source #1	Sampled sine (1 MHz)	10	10
Source #2	Sampled sine (2 MHz)	15	10
Source #3	Sampled sine (3 MHz)	30	10

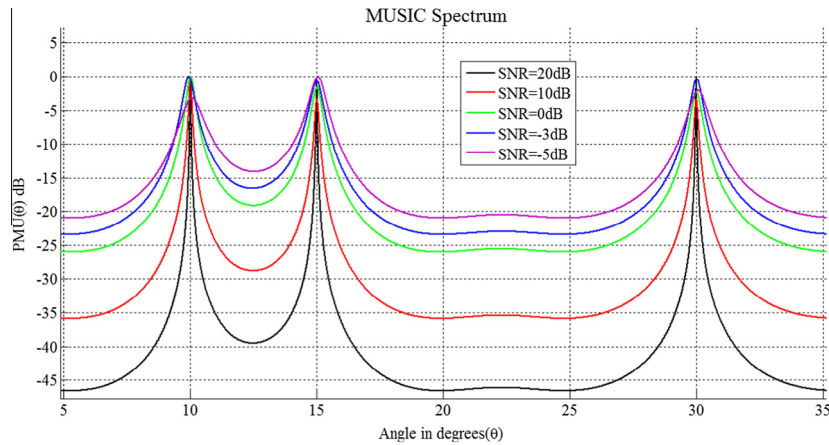


Fig. 14 MUSIC spectrum with changing “SNR”.

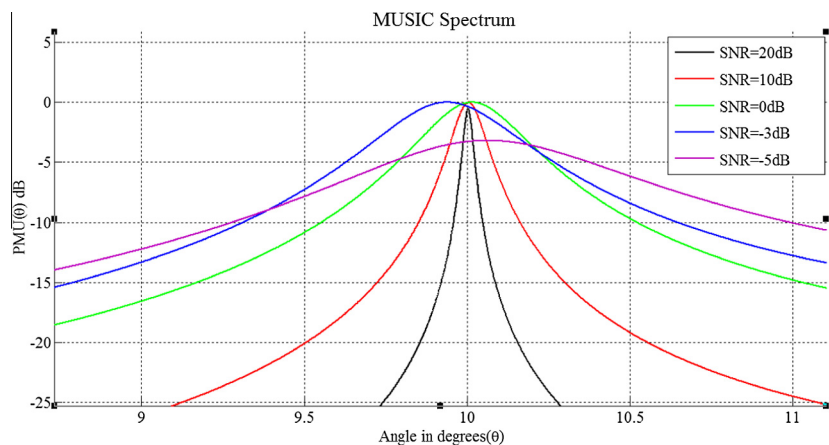


Fig. 15 Detailed view of Fig. 14 at $\theta = 10^\circ$.

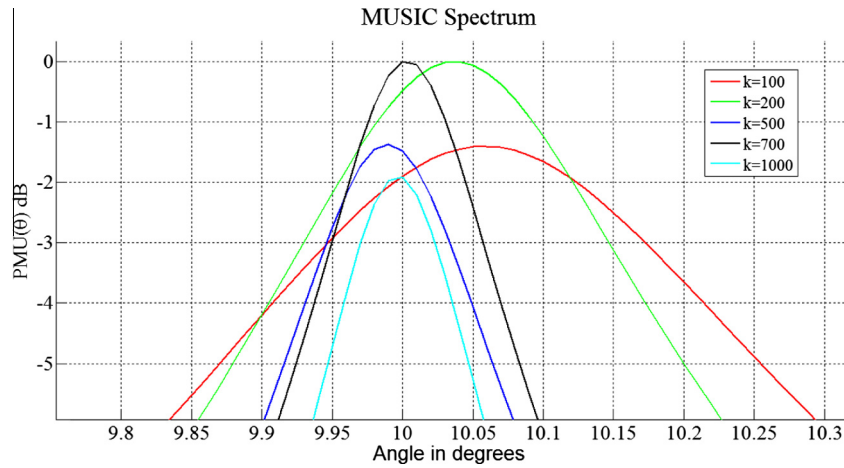


Fig. 16 MUSIC spectrum with changing “ k ”.

- If the elements are spaced closely (d decreases), the coupling effect will be larger. Therefore, the elements have to be far enough to avoid mutual coupling and the spacing has to be smaller to avoid grating lobes.

Applying the input data as shown in Table 4 to the simulator

F_s = sampling frequency = 10 MHz.

k = number of data samples = 500.

M = number of array elements = 20.

- As shown in Fig. 9 we can notice that for values of $d > 0.7\lambda$ grating lobes in negative side appear causing wrong estimation.
- As shown in Fig. 10 we can notice that for values of $d < 0.7\lambda$ as d increases the peaks become sharper and we can get high accuracy.
- As shown in Fig. 11 we can notice that the optimum value of inter-element spacing is $d = 0.6\lambda$.

5.3. MUSIC spectrum with changing “ d ” and higher angles of incident signals

Applying the data as shown in Table 5 to the simulator

F_s = sampling frequency = 10 MHz.

k = number of data samples = 500.

M = number of array elements = 20.

- As shown in Fig. 12 we face two challenges between increasing “ d ” to enhance the estimation accuracy and decreasing “ d ” to avoid the side lobes which will cause wrong estimation.
- So the optimum value of inter-element spacing using ULA antenna is “ $d = 0.55\lambda$ ” as shown in Fig. 13.

5.4. MUSIC spectrum with changing Signal-to-Noise Ratio “SNR”

Applying the data as shown in Table 6 to the simulator

F_s = sampling frequency = 10 MHz.

k = number of data samples = 500.

M = number of array elements = 20.

d = inter-element spacing = 0.55λ .

- As shown in Fig. 14, it is clear that as the signal power received is higher than noise power we can get high resolution.
- As shown in Fig. 15 bad estimation was marked when (SNR < 0 dB).

5.5. MUSIC spectrum with changing number of data samples “ k ”

Applying the data as shown in Table 7 to the simulator

F_s = sampling frequency = 10 MHz.

M = number of array elements = 20.

d = inter-element spacing = 0.55λ .

- As shown in Fig. 16, the larger the number of data samples taken, the better the quality will be.
- The optimum value for number of data samples ($k = 1000$).

6. Conclusions

The main motivation of this paper is the possibility of optimization of parameters that can affect the performance of AOA estimation using MUSIC algorithm. We can summarize the results as

1. The performance of MUSIC improves with more elements starting from $M = 10$, and good results especially in the case of adjacent users using $M \geq 20$.
2. As the number of data snapshots increases, the MSE (Mean Square Error) decreases, which results in a high detection accuracy of closely spaced signals ($k = 1000$).
3. As SNR increases, better accuracy we can get which means environment with little noise.
4. The most significant result in our study is: the existence of an optimal value for the uniform array spacing “ d ” (not necessarily half the wavelength), which exhibits the best estimation of the AOA (cf. Figs. 12 and 13).

5. The effect of this optimal value of “ d ” may be attributed to the minimization of the grating lobes of the antenna array at this special value of “ d ”, and hence an increased accuracy in the determination of the AOA is the direct consequence of such an optimal value. This optimization will be a major importance for closely spaced users, and could be very helpful when expanding the mobile services offered to increased number of users in a limited geographical area.

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